# Path Planning and Optimization for Cuspidal 6R Manipulators

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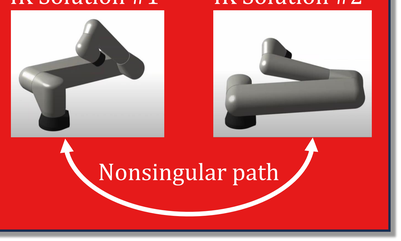
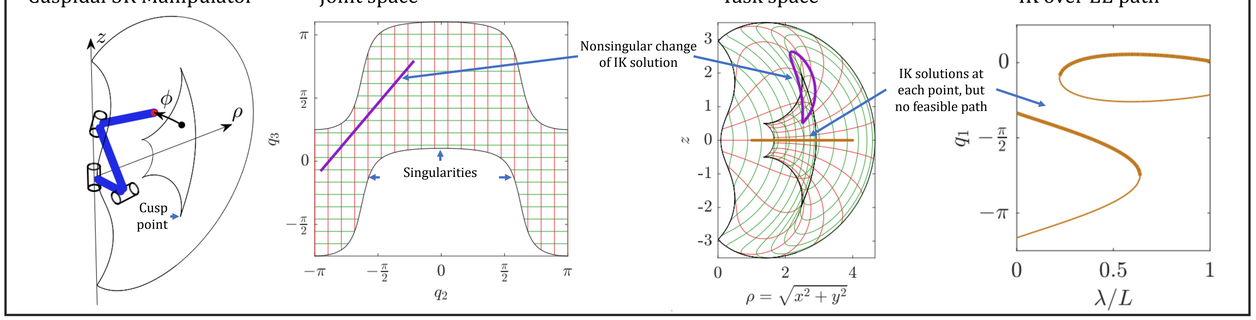
## Cuspidal robots switch IK solutions without crossing singularities

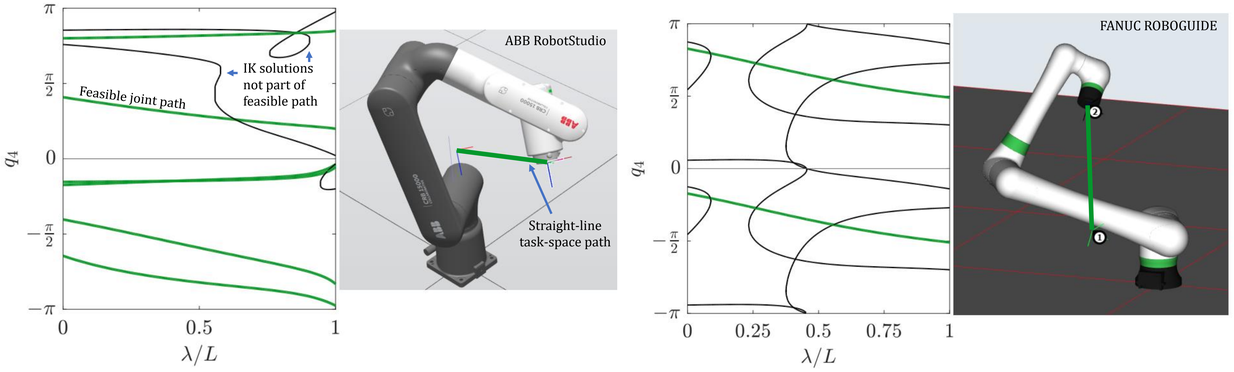
Classical path planners may fail




## 1. Task-space paths inside workspace may not have joint-space paths




## 2. Feasibility depends on choice of IK solution, but industrial simulators don't return all IK solutions[1]



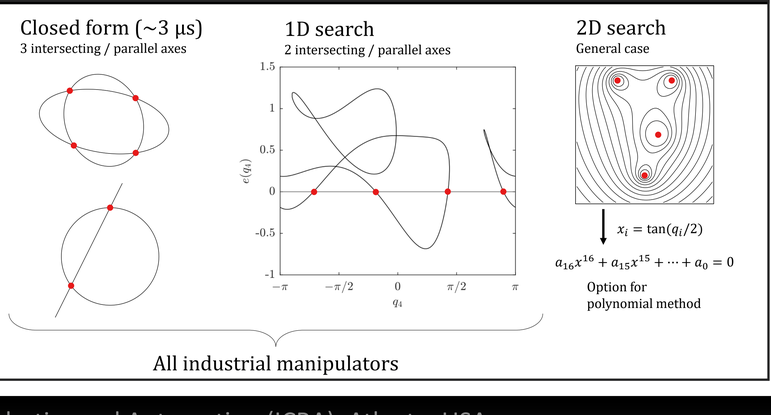

## 3. IK-Geo: Highly efficient solver finds all IK solutions for any robot using subproblem decomposition[2]

**Fastest solver** in our testing
- >40x faster IK for UR5 than IKFast
- Independently verified performance

**Robust** to singularities

**All solutions** returned
- Continuous approximate solutions
- Sometimes least-squares solutions

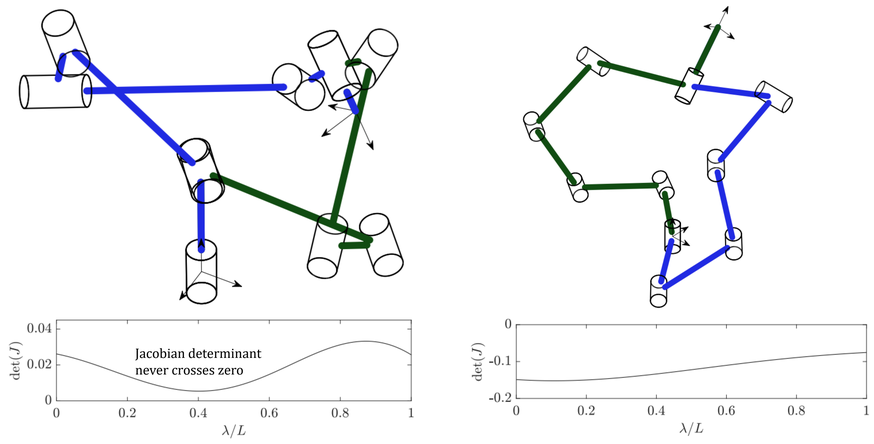
**All 6R robots** compatible
- Also 7R robots[3] and parallel robots




## 4. Found new cuspidal robots with efficient identification method[1]

**ABB GoFa is cuspidal**

**Robots with 3 parallel axes are cuspidal**




Robot cuspidal if algorithm terminates

While nonsingular path not found:
1. Pick a random EE pose
2. Find all IK solutions with IK-Geo
3. Check linear paths among solutions

Compared to previous methods:
- Simpler (no path optimization)
- Faster (using IK-Geo)

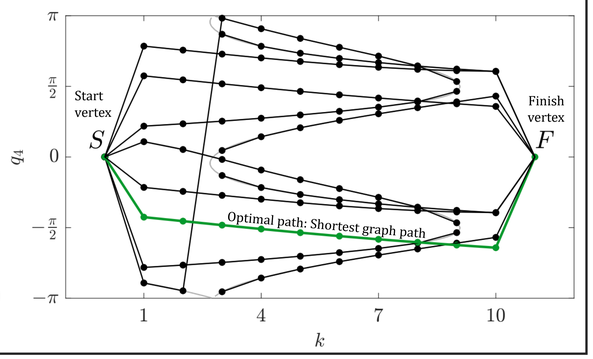
## 5. Graph-based planner finds optimal feasible joint path for a given task path[1]

Reduce to shortest path problem in directed acyclic graph (DAG):
1. Find all IK solutions for each sample with IK-Geo
2. Connect vertices based on threshold
3. Weigh edges based on incremental cost
4. Find shortest path

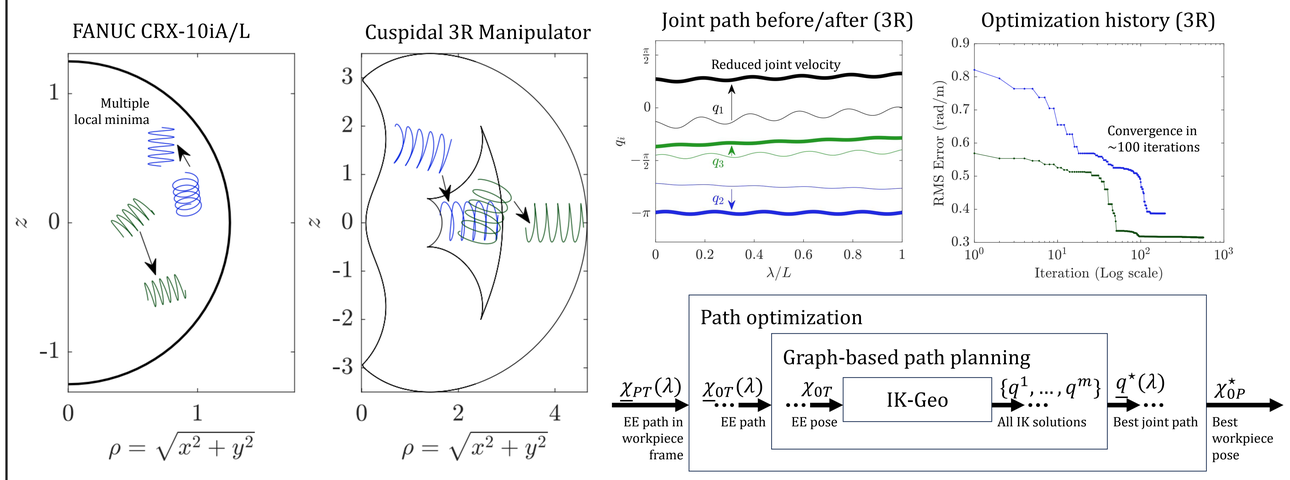
Simplest metric: Minimize joint velocity

$$C(\underline{q}(\lambda)) = \left\|\underline{q}'(\lambda)\right\|_2^2 = \int_0^L \left\|\frac{dq(\lambda)}{d\lambda}\right\|^2 d\lambda$$

Other options:
- Joint limit avoidance
- Singularity avoidance
- Repeatability (closed paths)




## 6. Path optimization finds best rigid-body offset of end effector path[1]




[1] A. J. Elias and J. T. Wen, "Path planning and optimization for cuspidal 6R manipulators," *arXiv preprint arXiv:2501.18505*, 2025.

[2] A. J. Elias and J. T. Wen, "IK-Geo: Unified robot inverse kinematics using subproblem decomposition," *Mech. Mach. Theory*, vol. 209, p. 105971, 2025.

[3] A. J. Elias and J. T. Wen, "Redundancy parameterization and inverse kinematics of 7-DOF revolute manipulators," *Mech. Mach. Theory*, vol. 204, p. 105824, 2024.